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# QoS-Driven Energy-Efficient Resource Allocation in Multiuser Amplify-and-Forward Relay Networks

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**Abstract**—In this paper, we investigate energy-efficient joint subcarrier pairing, subcarrier allocation, and power allocation algorithms for improving the network energy efficiency (EE) in multiuser amplify-and-forward (AF) relay networks while ensuring the desired quality-of-service (QoS) requirement for the users through the concept of “network price”. Further, we introduce a network price paid for the consumed power as a penalty for the achievable sum rate and formulate a resource allocation problem subject to limited transmit power budget and QoS constraints. The formulated problem is a non-convex binary mixed-integer non-linear programming (MINLP) problem and it is hard to solve the problem. We then apply a concave lower bound on the pricing-based network utility to transform the problem into a convex one. The dual decomposition method is adopted to propose a  $\mathcal{L}$ -price resource allocation algorithm to find the near-optimal solution. Next, we discuss the optimal utility-price from an EE perspective. Moreover, we rigorously analyze the behaviour of the network pricing-based resource allocation in two-user case under different noise operating regimes, and discuss the corresponding strategies for achieving energy-efficient transmission, generating water-filling and channel-reversal approaches. To strike a balance between the computational complexity and the optimality, we propose a low-complexity suboptimal algorithm. Furthermore, we extend the proposed algorithm to maximize the EE of multiuser multi-relay full-duplex (FD) relay networks and the relay networks with an eavesdropper. The performance gain of the proposed algorithms is validated through computer simulations.

**Index Terms**—Resource allocation, quality-of-service, energy efficiency, multiuser, amplify-and-forward, relay networks.

## I. INTRODUCTION

In recent years, cooperative communication has emerged as a promising way to enhance the reliability, coverage and performance of wireless communication systems [1]. The rapid growth of Internet-of-Things (IoT) in cooperative wireless communication has received considerable attention from the research community due to the hike in power dissipation costs, ecological, and environmental reasons, to emphasize on green wireless communications [2],[3]. Although nodes in cooperative communications are often low-powered, they are typically powered by batteries, resulting in limited operating time.

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Frequent battery replacement is thus required for continuous operation of the nodes, which is difficult to change or recharge. As a result, a finite capacity of batteries restrains the network performance of cooperative wireless networks. However, the lifespan of a cooperative network can be increased by minimizing the energy consumption in the network. Moreover, the efficient power utilization enables us to depreciate the carbon footprint, thereby offering a green solution. Thus, the energy efficiency (EE), defined as the number of bits transmitted per unit of energy, has become an important metric for the next-generation wireless communication systems.

Various relaying schemes have been proposed for cooperative communications, like amplify-and-forward (AF) and decode-and-forward (DF) [4]. In the former one, the relay retransmits the amplified signal to the destination nodes, whereas in the latter one, the relay attempts to decode the received signals and retransmit the re-encoded information bits to the destination nodes. Since the conventional DF scheme suffers from decoding errors, it can perform better than the AF scheme only if an appropriate mechanism is encompassed to avoid the problem of error propagation, e.g., forwarding signals only at instantaneous high SNR as in [5] and/or adopting error control codes as in [6]. This problem becomes more challenging for the DF scheme in multiple-input multiple-output (MIMO) relay channels for which the source-to-relay link quality is dominated by the interference terms, and it thus requires stronger error control codes with complex decoding/encoding processing at the relays. Although a very long code provides high error correction capability, the decoder complexity, which increases with the code length, accounts for a significant portion compared with other baseband pre-processing, e.g., equalization. The complexity issues of the decoder design for the DF scheme are discussed in [7]. In contrast, the AF scheme, which does not require decoding/re-encoding at the relays, offers a viable strategy with modest computational burden, whilst achieving considerable performance gains. This benefit is even more attractive in MIMO relay networks, in which decoding multiple data streams can be computationally strenuous, and thus we will focus on designing of the energy-efficient AF scheme in this paper. Furthermore, the resource allocation in a relay network that maximizes the spectral efficiency (SE) while utilizing the minimum power and simultaneously maintaining the desired QoS becomes a challenging issue [8]. In brief, the main objective of this work is to study the problem of QoS-based joint subcarrier pairing, subcarrier and power allocation in multiuser interference networks for improving the energy utilization among users.

The optimization of power usage in multiuser relay net-

works is quintessential not only because of energy dissipation and system throughput, but also due to the interference management. Further, in a wireless network there are two main sources of power dissipation, firstly, the transmit power by nodes allocated in response to the instantaneous channel conditions, that indeed remains dynamic in nature, and secondly, static power dissipation, which is the power utilized in various activities, like signal processing, battery backup and site cooling [9]. Clearly, the static power of the network remains constant in the model, therefore, the relay network performance mainly depends on the transmit power of the users and the relay node and their corresponding channel conditions. Thus, by allocating transmit power to all the source and relay nodes according to the respective instantaneous channel characteristics, the network's performance can be meliorated significantly. A plethora of works on multiuser relay networks mainly focused on power allocation from the perspective of throughput maximization [10], QoS enhancement [11], user selection and coverage expansion [12]. However, these existing research works have not focused on designing energy-efficient power allocation problem in multiuser scenarios. Motivated from above discussion, we will focus on how to maximize the average EE of multiuser relay network by joint optimization of resource allocation among the users.

The optimal power allocation scheme that maximizes the ergodic achievable rates was investigated in [13] for a multi-pair massive MIMO two-way AF relaying with imperfect channel state information (CSI). In [14], the problem of the joint resource allocation for uplink coordinated multipoint transmission/reception (CoMP) with limited backhaul link was studied under the compress-and-forward scheme, wherein a central backhaul node processes user pairing and subcarrier mapping, but this work is limited to uplink multiple access (MA) phase only, and thus it cannot be directly applied to multi-hop scenarios specially when a set of users are active in uplink MA phase while other set of users are active in downlink broadcast (BC) phase, whilst the users of the MA phase are willing to communicate with the corresponding users in the BC phase through an AF relay node that can also be considered as a low power base station (BS). The power allocation strategies with subcarrier pairing were proposed in [15]–[19] for the relay systems. The joint subcarrier pairing and power allocation schemes were investigated in [15] for a single user pair OFDM DF relay systems in order to improve the capacity, while the joint optimization of subcarrier pairing, relay selection and power allocation was studied in [16] for OFDM DF multi-relay networks. Joint optimization of subcarrier pairing and power allocation scheme was proposed in [18] under the total network power constraint or the individual power constraint on each node, wherein each subcarrier pair is assigned to only one relay in order to avoid interference among all the relays and the destination node receives signals from only one relay. The resource allocation schemes in [15]–[19] cannot be easily applied when multiple user pairs exist in the network, where each subcarrier pair is required to be assigned to an individual user pair, and the relay node operates in an AF mode. Additionally, the existing works in [15]–[19] have not focused on joint optimization of power allocation, subcarrier

pairing and subcarrier allocation with QoS requirement for multiuser AF relay network scenario from an EE perspective.

When the network EE is adopted as the objective function, the subcarrier pairing, subcarrier and power allocation schemes cannot be directly applied. In fact, there are only a few works that have considered the EE as a key metric for designing the optimal power allocation policies in relay networks [8], [20]–[26]. The optimal power allocation policy for multiuser two-way relay networks was studied in [8] while ensuring the QoS. The authors in [20] focused on pricing-based power allocation schemes in multiuser AF relay networks. Only power allocation policies were investigated in [8] and [20],[21]. An energy-efficient resource allocation scheme for AF cooperative OFDMA networks was studied in [22] for higher SNR-region without considering the user's QoS requirement, while the resource allocation scheme for multiuser downlink OFDMA cellular networks with user cooperation was proposed in [23] to maximize the EE under a very strong assumption of high signal-to-noise ratio (SNR) regime, which is not possible in the practical scenario. The resource allocation problem for EE-SE tradeoff was studied in [24] for a single-link OFDM wireless system. An energy-efficient resource scheduling algorithm for downlink transmission in multiuser OFDMA networks was investigated in [25] under imperfect CSI, whereas the work [25] has been extended in [26] for multicarrier under perfect CSI knowledge. The resource allocation problem in [25] and [26] was optimized only in downlink scenario for maximizing EE. The problem of energy-efficient joint optimization of the power allocation, subcarrier pairing and subcarrier allocation with QoS requirement for multiuser AF relay networks has not been well investigated in the literature.

Unlike the previous existing research works [14]–[19], wherein the throughput in OFDM network was maximized by optimizing either of the following: *i*) subcarrier allocation among different users, *ii*) subcarrier pairing at relay node, where the signal received at relay over one subcarrier is re-transmitted on a different subcarrier, *iii*) power allocation over different subcarriers at each transmitting node; or *iv*) power allocation and subcarrier assignment, and [20]–[26], in this paper, we propose a unified energy-efficient resource allocation scheme by considering subcarrier permutation, power optimization, and subcarrier allocation all together and believe that we have made significant contribution in designing of energy-efficient subcarrier pairing, subcarrier and power allocation in multi-user multicarrier AF relaying networks. This is the first work that investigates the energy-efficient resource allocation algorithm through the concept of “network price”, which enables us to strike a balance between the achievable sum rate and the total power consumption in the relay networks. The terms ‘penalty’ and ‘price’ are used in an essentially interchangeable form. The main contributions in this work are as follows. A network pricing-based approach is adopted for the considered multiuser AF relay networks in order to achieve an energy-efficient communication. Through a joint subcarrier pairing, subcarrier and power allocation, we intend to maximize the pricing-based network utility function in multiuser multicarrier relay network subject to a total transmit, subcarrier pairing, and subcarrier allocation constraints. The

formulated primal problem is a non-convex MINLP problem that is NP-hard to solve. To make the problem tractable, we adopt a successive convex approximation (SCA) approach, for which the objective function is lower bounded by a concave function, and a series of transformations. Then, based on the concepts of dual decomposition, a utility-based joint subcarrier pairing, subcarrier and power allocation algorithm is proposed for iteratively improving the lower bound and attain the near-optimal solution. We then discuss the optimal network price from an EE perspective, and then an iterative EE maximization algorithm is proposed to iteratively find the maximum EE in terms of optimal network price. To get more insights into the proposed approach, we rigorously analyze the behavior of the network pricing-based resource allocation in two-user case under different noise operating regimes, and discuss the corresponding strategies for achieving energy-efficient transmission. To strike a balance between the computational complexity and the optimality, we propose a low-complexity suboptimal algorithm. Furthermore, we extend the proposed resource allocation algorithm to maximize the EE of multiuser AF relay networks with two more practical models, first multi-relay nodes operating in full-duplex mode, and; secondly, with an additional eavesdropper relay node. The performance of the proposed iterative resource allocation and suboptimal algorithms are evaluated and validated by computer simulations. Additionally, we also demonstrate the impact of the EE on the SE under various network parameters such as number of subcarriers and number of users.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, we introduce a network utility function and formulate the joint optimization problem as a MINLP problem, followed by the procedure of transforming the non-convex problem into a convex one. An iterative EE resource allocation algorithm is proposed in Section IV. The suboptimal algorithm is presented in Section V. We analyze the resource allocation algorithm for two-user case under different noise regimes and the complexity of proposed and standard algorithms in Section VI. The extension of the design framework is illustrated in Section VII. Numerical results are given in Section VIII. Finally, conclusions and future directions are drawn in Section IX.

## II. SYSTEM AND POWER DISSIPATION MODEL

### A. System Model

We consider a multiuser AF relay network with  $N_{sc}$  subcarriers as shown in Fig. 1, where there are one relay node  $R$  and a set  $\mathcal{N} = (1, 2, \dots, N)$  source-destination pairs (i.e., users). It is assumed that all nodes are equipped with a single antenna. Further, the relay node has perfect knowledge of channel state information (CSI). For simplicity, it is assumed that there is no direct link between the source and the destination nodes due to the path loss and large-scale fading. Furthermore, in this practical network, the relay network is operated in half-duplex mode with two transmission phases: the MA phase and the BC phase.

In the MA phase,  $N$  source nodes ( $S_n$ ,  $n \in \mathcal{N}$ ) concurrently transmit signals to the relay node  $R$ , and the received

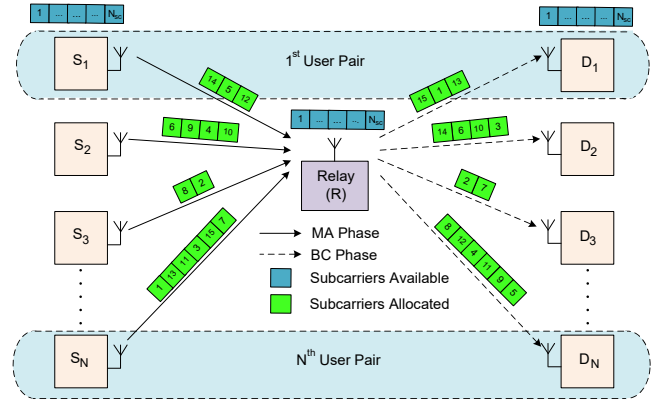


Fig. 1. A dual-hop multiuser AF relay network.

signal at the relay node on the  $j$ -th subcarrier can be given by

$$y_R^{(j)} = \sum_{n=1}^N h_{S_n}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)} + n_R^{(j)}, \quad (1)$$

where  $h_{S_n}^{(j)}$  represents the channel coefficient from the  $n$ -th source node to the relay node on the  $j$ -th subcarrier,  $x_{S_n}^{(j)}$  denotes the transmitted signal of the  $n$ -th source node on the  $j$ -th subcarrier with unit transmit power, i.e.,  $\mathbb{E} |x_{S_n}^{(j)}|^2 = 1$ ,  $n_R^{(j)}$  represents the additive white Gaussian noise (AWGN) at the relay node on the  $j$ -th subcarrier with zero mean and variance  $\mathbb{E} [n_R^{(j)}]^2 = \sigma_R^{(j)2}$ , and  $P_{S_n}^{(j)}$  indicates the transmit power of the  $n$ -th user on the  $j$ -th subcarrier.

In the BC phase, the relay node amplifies the received signal with a normalized amplifying factor, expressed as

$$\rho_R^{(k)} = \sqrt{\frac{P_R^{(k)}}{\sum_{n=1}^N P_{S_n}^{(j)} |h_{S_n}^{(j)}|^2 + \sigma_R^{(j)2}}}, \quad (2)$$

where  $P_R^{(k)}$  represents the transmit power of the relay node on the  $k$ -th subcarrier. Thus, the transmitted signal from the relay node to the destination nodes on the  $k$ -th subcarrier is

$$x_R^{(k)} = \rho_R^{(k)} y_R^{(j)} = \rho_R^{(k)} \sum_{n=1}^N h_{S_n}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)} + \rho_R^{(k)} n_R^{(j)}, \quad (3)$$

The received signal at the  $n$ -th destination node on the  $k$ -th subcarrier can be written as

$$\begin{aligned} y_{RD_n}^{(k)} &= g_{RD_n}^{(k)} x_R^{(k)} + n_{D_n}^{(k)}; \\ &= \underbrace{g_{RD_n}^{(k)} \rho_R^{(k)} h_{S_n}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)}}_{\text{Desired Signal}} + \underbrace{g_{RD_n}^{(k)} \rho_R^{(k)} \sum_{l=1, l \neq n}^N h_{S_l}^{(j)} \sqrt{P_{S_l}^{(j)}} x_{S_l}^{(j)}}_{\text{Interference}} \\ &\quad + \underbrace{g_{RD_n}^{(k)} \rho_R^{(k)} n_R^{(j)} + n_{D_n}^{(k)}}_{\text{Compound Noise}}, \end{aligned} \quad (4)$$

where  $g_{RD_n}^{(k)}$  represents the channel coefficient from the relay node to the  $n$ -th destination node  $D_n$  on subcarrier  $k$  and  $n_{D_n}^{(k)}$



denotes the AWGN at destination  $D_n$  on subcarrier  $k$  with noise variance  $\mathbb{E} \left[ \left| n_{D_n}^{(k)} \right|^2 \right] = \sigma_{D_n}^{(k)2}$ , respectively. Further, the SINR at destination  $D_n$  can be denoted as in (5), shown on the top of next page.

Define  $\Lambda^{j,k} \in \{0,1\}$  as a subcarrier pairing indicator variable signifying that  $\Lambda^{j,k} = 1$  if the  $j$ -th subcarrier in the MA phase is paired with the  $k$ -th subcarrier in BC phase and  $\Lambda^{j,k} = 0$  otherwise. Further, we define binary variables  $\Omega_n^{(j,k)} \in \{0,1\}$  as a subcarrier allocation variable such that  $\Omega_n^{(j,k)} = 1$  if  $(j,k)$ -th subcarrier is allocated to the  $n$ -th user pair while  $\Omega_n^{(j,k)} = 0$  otherwise. From the capacity formula and using the subcarrier pairing and allocation variables, the achievable sum rate for the  $n$ -th user pair can be written as

$$\mathcal{R}_n = \frac{1}{2} \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \log_2 \left( 1 + \Upsilon_n^{(j,k)} \right), \quad (6)$$

where  $\mathbf{P} = \{P_{S_n}^{(j)}\}$ ,  $\mathbf{P}_R = \{P_R^{(k)}\}$ ,  $\mathbf{\Lambda} = \{\Lambda^{j,k}\}$ , and  $\mathbf{\Omega} = \{\Omega_n^{(j,k)}\}$ ,  $\forall n, j, k$ . The factor  $1/2$  comes from the fact that transmission takes place in two-hops. The achievable sum rate of the network is then calculated by summing up all users' sum rate as follows:

$$\begin{aligned} \mathcal{R}_T &= \sum_{n=1}^N \mathcal{R}_n(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) \\ &= \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \log_2 \left( 1 + \Upsilon_n^{(j,k)} \right), \end{aligned} \quad (7)$$

### B. Power Dissipation Model

By proper utilization of the available power we can maximize the EE of the network. The power dissipation takes place in various forms, mainly categorized into 1) transmit power, 2) processing power; and 3) circuit power, respectively. The transmit power is utilized for transmitting signal from one node to another and it directly depends on the external factors such as channel conditions, cell coverage areas and thus, it varies with each node and subcarrier whereas the processing and circuit power consumption is directly commensurate to the energy consumed while processing the signal by the node using various circuitry components like analog-to-digital converter (ADC) etc, thus it remains static for each node but it directly varies with the number of antennas. The total power dissipation in the network can therefore be given as [20]

$$\begin{aligned} P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) &= \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \underbrace{\Lambda^{j,k} \Omega_n^{(j,k)} \left( P_{S_n}^{(j)} + P_R^{(k)} \right)}_{\text{Dynamic Power Dissipation} \leq P_{max}} \\ &\quad + \underbrace{(2N+1)P^C + (N+1)Q^C}_{\text{Static Power Dissipation, } X^C \geq 0}, \end{aligned} \quad (8)$$

where  $P^C$  and  $Q^C$  represent the circuit and processing power dissipation per antenna at each node, respectively.

### III. NETWORK UTILITY FUNCTION AND PROBLEM FORMULATION

Using (7) and (8), we define the network utility function as

$$\mathcal{U}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) = \mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) - \mathcal{L}P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}), \quad (9)$$

where the second term in (9) i.e.,  $\mathcal{L}P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})$  denotes maximum penalty/price paid by the users, wherein  $\mathcal{L}$  represents the unit price of resources, i.e., power and subcarriers. It is clear that when  $\mathcal{L} = 0$ , the resource allocation problem becomes a sum rate maximization problem where each user utilizes maximum resources in order to increase the sum rate for their own, whereas when the value of  $\mathcal{L}$  increases, it shows the importance of spectrum and power resources for the design of joint subcarrier pairing, subcarrier and power allocation in the relay networks. However, when  $\mathcal{L} \rightarrow \infty$ , no transmission is good enough for maximizing the network's utility.

Our goal is to maximize the network utility function defined in (9), subject to the following constraints: 1) limited total transmit power budget, 2) promising QoS requirement for each user, 3) one-to-one subcarrier pairing in both hops; and 4) subcarrier allocation to user pairs. Consequently, the optimization problem for the joint subcarrier and power allocation in QoS-promising AF relay networks can be formulated as

$$\begin{aligned} (\text{OP1}) \quad & \max_{\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \mathcal{U}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) \\ \text{s.t.} \quad & (C.1) \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( P_{S_n}^{(j)} + P_R^{(k)} \right) \leq P_{max}; \\ & (C.2) \Upsilon_n^{(j,k)} \geq \Upsilon_{n,min}, \quad \forall n, j, k; \\ & (C.3) \sum_{j=1}^{N_{sc}} \Lambda^{j,k} = 1, \quad \forall k; \\ & (C.4) \sum_{k=1}^{N_{sc}} \Lambda^{j,k} = 1, \quad \forall j; \\ & (C.5) \sum_{n=1}^N \Omega_n^{(j,k)} = 1, \quad \forall j, k; \\ & (C.6) \Lambda^{j,k} \in \{0,1\}, \Omega_n^{(j,k)} \in \{0,1\}, \quad \forall n, j, k; \\ & (C.7) P_{S_n}^{(j)} \geq 0, P_R^{(k)} \geq 0, \quad \forall n, j, k, \end{aligned} \quad (10)$$

where  $\Upsilon_{n,min}$  denotes the minimum SINR requirement for the  $n$ -th user in the network. Physically, the constraint (C.1) ensures that the sum of the power allocated to all users and the relay node cannot exceed the maximum transmit power budget  $P_{max}$ , the constraint (C.2) guarantees the minimum QoS requirement for each user, while the constraints (C.3) and (C.4) mandate that any subcarrier in the first hop is paired with one subcarrier in the second hop, and vice versa; and the constraint (C.5) guarantees that each subcarrier pair is allocated to only one user pair.

#### A. Convexification of Non-convex Optimization Problem

The primal optimization problem (OP1) is a MINLP problem, and thus it is non-convex and intractable [30]. In

$$\Upsilon_n^{(j,k)} = \frac{P_R^{(k)} P_{S_n}^{(j)} |g_{RD_n}^{(k)} h_{S_n}^{(j)}|^2}{P_R^{(k)} |g_{RD_n}^{(k)}|^2 \sum_{l=1, l \neq n}^N P_{S_l}^{(j)} |h_{S_l}^{(j)}|^2 + P_R^{(k)} |g_{RD_n}^{(k)}|^2 \sigma_R^{(j)^2} + \sigma_{D_n}^{(k)^2} \left( \sum_{n=1}^N P_{S_n}^{(j)} |h_{S_n}^{(j)}|^2 + \sigma_R^{(j)^2} \right)}, \quad (5)$$

this subsection, convexification strategies for the non-convex problem (OP1) are introduced and discussed. Through SCA method, a lower bound maximization problem can be formulated as

$$\begin{aligned} (\text{OP2}) \quad & \max_{\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \mathcal{U}_{LB}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta) \\ \text{s.t.} \quad & (C.1) - (C.7), \end{aligned} \quad (11)$$

where  $\mathcal{U}_{LB}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta)$  is a lower bound on  $\mathcal{U}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})$  and is defined as in (12), shown on the top of next page, where  $\alpha = \{\alpha_n^{(j,k)}\}$  and  $\beta = \{\beta_n^{(j,k)}\}$ . The coefficients  $\alpha_n^{(j,k)}$  and  $\beta_n^{(j,k)}$  can be determined as follows:

$$\alpha_n^{(j,k)} = \frac{\varsigma_n^{(j,k)}}{1 + \varsigma_n^{(j,k)}}; \quad (13)$$

$$\beta_n^{(j,k)} = \log_2(1 + \varsigma_n^{(j,k)}) - \alpha_n^{(j,k)} \log_2(\varsigma_n^{(j,k)}), \quad (14)$$

for any given  $\varsigma_n^{(j,k)} > 0$ . Note that the equality in (12) holds only if  $\alpha_n^{(j,k)} = \Upsilon_n^{(j,k)} \left(1 + \Upsilon_n^{(j,k)}\right)^{-1}$  and  $\beta_n^{(j,k)} = \log_2(1 + \Upsilon_n^{(j,k)}) - \alpha_n^{(j,k)} \log_2(\Upsilon_n^{(j,k)})$ . When  $\Upsilon_n^{(j,k)}$  approaches positive infinity,  $(\alpha_n^{(j,k)}, \beta_n^{(j,k)}) = (1, 0)$ . However, the optimization problem (OP2) is still non-convex and hence we introduce the following auxiliary power variables,  $\hat{P}_{S_n}^{(j)} = \log P_{S_n}^{(j)}$  and  $\hat{P}_R^{(k)} = \log P_R^{(k)}$ . The auxiliary relaxed lower bound optimization problem is written as

$$\begin{aligned} (\text{OP3}) \quad & \max_{\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \mathcal{U}_{LB}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta) \\ \text{s.t.} \quad & (C.1) \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}} \right) \leq P_{max}; \\ & (C.2) \ln \hat{\Upsilon}_n^{(j,k)} \geq \ln \Upsilon_{n,min}, \quad \forall n, j, k; \\ & (C.3) - (C.6); \\ & (C.7) e^{\hat{P}_{S_n}^{(j)}} \geq 0, e^{\hat{P}_R^{(k)}} \geq 0, \quad \forall n, j, k, \end{aligned} \quad (15)$$

where  $\hat{\mathbf{P}}$  is the  $N \times N_{sc}$  matrix of auxiliary users power allocation,  $\hat{\mathbf{P}}_R$  presents  $N_{sc} \times 1$  vector of auxiliary relay power allocation, respectively, and  $\hat{\Upsilon}_n^{(j,k)}$  is defined as in (16), shown on the top of next page.

*Lemma 1:* For any given  $\alpha_n^{(j,k)}, \beta_n^{(j,k)}, \mathcal{L}$  and fixed subcarrier pairing  $\mathbf{\Lambda}$ , and subcarrier allocation  $\mathbf{\Omega}$ , the lower bound  $\mathcal{U}_{LB}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta)$  is concavified by the change of variables  $\hat{P}_{S_n}^{(j)} = \log P_{S_n}^{(j)}$  and  $\hat{P}_R^{(k)} = \log P_R^{(k)}$ .

*Proof:* Substituting  $\hat{P}_{S_n}^{(j)} = \log P_{S_n}^{(j)}$  and  $\hat{P}_R^{(k)} = \log P_R^{(k)}$  into (12), the lower bound becomes

$$\begin{aligned} \mathcal{U}_{LB}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta) = & \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left[ \alpha_n^{(j,k)} \log_2(\hat{\Upsilon}_n^{(j,k)}) + \beta_n^{(j,k)} \right] \\ & - \mathcal{L} \left( \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}} \right) + X^C \right), \end{aligned} \quad (17)$$

where  $\ln(\hat{\Upsilon}_n^{(j,k)})$  can be expanded as in (18). From (17) and (18), we know that for any given  $\alpha_n^{(j,k)}, \beta_n^{(j,k)}, \mathcal{L}$  and fixed subcarrier pairing  $\mathbf{\Lambda}$ , and subcarrier allocation  $\mathbf{\Omega}$ , the lower bound  $\mathcal{U}_{LB}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \alpha, \beta)$  contains the summation of linear terms and concave terms, particularly log-sum-exp functions and minus-exp functions, and thus justifying the concavity-nature of the lower bound of the network utility function. ■

#### IV. EE RESOURCE ALLOCATION ALGORITHM

In this section, a joint subcarrier pairing, subcarrier allocation, and power allocation optimization problem is formulated for multiuser relay interference network from viewpoint of EE maximization. Since the optimization problem (OP3) is a MINLP problem, we can find the optimal resource allocation solution through an exhaustive search over all variables [29]. However, the computational complexity of an exhaustive search method is very high, specially for higher number of  $N_{sc}$ . The optimization problem (OP3) is a convex problem for fixed subcarrier pairing and allocation and coefficients  $\{\alpha_n^{(j,k)}, \beta_n^{(j,k)}\}$ , and thus it can be solved by employing a dual decomposition method<sup>1</sup> [30].

The Lagrangian function for the problem (OP3) can be written as

$$\begin{aligned} \mathcal{L}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \mu, \nu) = & \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left[ \alpha_n^{(j,k)} \log_2(\hat{\Upsilon}_n^{(j,k)}) + \beta_n^{(j,k)} \right] \\ & - \mathcal{L} \left( \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}} \right) + X^C \right) \\ & - \mu \left( \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}} \right) - P_{max} \right) \\ & - \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \nu_n^{(j,k)} \left( \ln(\Upsilon_{n,min}) - \ln(\hat{\Upsilon}_n^{(j,k)}) \right), \end{aligned} \quad (19)$$

<sup>1</sup>When  $N_{sc} \rightarrow \infty$  the duality gap between the original problem and the dual problem tends to zero.

$$\begin{aligned} \mathcal{U}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) &\geq \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left[ \alpha_n^{(j,k)} \log_2 \left( \Upsilon_n^{(j,k)} \right) + \beta_n^{(j,k)} \right] - \mathcal{L} P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}); \\ &\triangleq \mathcal{U}_{LB}(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \boldsymbol{\alpha}, \boldsymbol{\beta}), \end{aligned} \quad (12)$$

$$\hat{\Upsilon}_n^{(j,k)} = \frac{e^{\hat{P}_R^{(k)} + \hat{P}_{S_n}^{(j)}} \left| g_{RD_n}^{(k)} h_{S_n}^{(j)} \right|^2}{\left| g_{RD_n}^{(k)} \right|^2 e^{\hat{P}_R^{(k)}} \sum_{l=1, l \neq n}^N e^{\hat{P}_{S_l}^{(j)}} \left| h_{S_l}^{(j)} \right|^2 + e^{\hat{P}_R^{(k)}} \left| g_{RD_n}^{(k)} \right|^2 \sigma_R^{(j)^2} + \sigma_{D_n}^{(k)^2} \left( \sum_{n=1}^N e^{\hat{P}_{S_n}^{(j)}} \left| h_{S_n}^{(j)} \right|^2 + \sigma_R^{(j)^2} \right)} \quad (16)$$

$$\begin{aligned} \ln \left( \hat{\Upsilon}_n^{(j,k)} \right) &= \hat{P}_R^{(k)} + \hat{P}_{S_n}^{(j)} + \ln \left| g_{RD_n}^{(k)} h_{S_n}^{(j)} \right|^2 \\ &\quad - \ln \left( \left| g_{RD_n}^{(k)} \right|^2 e^{\hat{P}_R^{(k)}} \sum_{l=1, l \neq n}^N e^{\hat{P}_{S_l}^{(j)}} \left| h_{S_l}^{(j)} \right|^2 + e^{\hat{P}_R^{(k)}} \left| g_{RD_n}^{(k)} \right|^2 \sigma_R^{(j)^2} + \sigma_{D_n}^{(k)^2} \left( \sum_{n=1}^N e^{\hat{P}_{S_n}^{(j)}} \left| h_{S_n}^{(j)} \right|^2 + \sigma_R^{(j)^2} \right) \right), \end{aligned} \quad (18)$$

where  $\mu$  and  $\boldsymbol{\nu} = \{\nu_n^{(j,k)}\}$  are the Lagrangian multipliers associated with the constraints (C.1) and (C.2), respectively. The dual Lagrangian function can be readily expressed as

$$g(\mu, \boldsymbol{\nu}) \triangleq \max_{\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \mathcal{L}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \mu, \boldsymbol{\nu}), \quad (20)$$

and the dual optimization problem is given by

$$\begin{aligned} \min_{\mu, \boldsymbol{\nu} \geq 0} g(\mu, \boldsymbol{\nu}) &= \min_{\mu, \boldsymbol{\nu} \geq 0} \max_{\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \mathcal{L}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}, \mathbf{\Omega}, \mu, \boldsymbol{\nu}) \\ \text{s.t.} \quad & (C.3) - (C.5), \end{aligned} \quad (21)$$

The dual problem in (21) can be decomposed into a master problem and a subproblem, and it can be solved in an iterative manner. The power allocation, subcarrier pairing and allocation variables  $\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \mathbf{\Lambda}$ , and  $\mathbf{\Omega}$  are obtained by solving a subproblem and then the Lagrange multipliers  $\mu, \boldsymbol{\nu}$ , are updated by solving the master problem for the obtained resource allocation. This process continues until convergence or satisfying the constraints<sup>2</sup>.

#### A. Solving the Subproblem

For fixed network price  $\mathcal{L}$ , the solutions of the subproblem can be obtained in two steps:

- 1) Solving the subproblem to find the source and the relay power allocation  $(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R)$  for fixed subcarrier pairing and allocation variables  $(\mathbf{\Lambda}, \mathbf{\Omega})$ .
- 2) To obtain subcarrier pairing and allocation  $\mathbf{\Lambda}$ , and  $\mathbf{\Omega}$ , we solve the subproblem for obtained power allocation  $(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R)$  in the first step.

1) *Power Allocation Solution:* With KarushKuhnTucker (KKT) conditions, for fixed subcarrier pairing and allocation matrices  $(\mathbf{\Lambda}, \mathbf{\Omega})$ , we can find the optimal power allocation solution at the  $(u+1)$ -th iteration by taking the partial derivative

<sup>2</sup>The iterative algorithm maximizes the lower bound maximization problem (OP3) for fixed coefficients  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  whereas the update of these coefficients guarantees a monotonic increase in the lower bound performance.

of (19) with respect to  $\hat{P}_{S_n}^{(j)}$  and  $\hat{P}_R^{(k)}$  and setting the gradient to zero, leading to equations (22) and (23), shown on the top of next page. To get more insights into the optimal power allocation, we further simplify the optimal solution through a linear approximation method [32], i.e.,  $\sqrt{a+b} \approx \sqrt{a} + \frac{1}{2\sqrt{a}}b$ . Due to subcarrier pairing and allocation, the value of interference term in (22) and (23) becomes zero, and thus the power allocation at the  $(u+1)$ -th iteration can be updated as follows:

$$\hat{P}_{S_n}^{(j)}(u+1) = \ln \left( \frac{\frac{\alpha_n^{(j,k)}}{2 \ln(2)} \sigma_R^{(j)^2} + \nu_n^{(j,k)} \sigma_R^{(j)^2}}{\mathcal{L} + \mu} \right); \quad (24)$$

$$\hat{P}_R^{(k)}(u+1) = \ln \left( \frac{\frac{\alpha_n^{(j,k)}}{2 \ln(2)} \sigma_{D_n}^{(k)^2} + \nu_n^{(j,k)}}{\mathcal{L} + \mu} \right), \quad (25)$$

From (24) and (25), we can observe that the power allocation policy depends not only on the Lagrangian multiplier  $\mu$ , but also on the network price  $\mathcal{L}$ . Further, when  $\mathcal{L} = 0$ , the power allocation only rely on the Lagrangian multipliers as in the case of sum rate maximization. Here, the inverse of the Lagrangian multiplier  $\mu$  plus the network price  $\mathcal{L}$  can be considered as a water-filling level which has to be chosen to meet the total transmit power budget  $P_{max}$ . However, in the case of without subcarrier pairing and subcarrier allocation, the power allocation policy also depends on the interference power among users. The  $n$ -th user has the ability to increase its power when the interference power generated by all other nodes to node  $n$  is small.

2) *Solution for Subcarrier Pairing and Allocation:* To derive the subcarrier pairing and allocation matrices  $\mathbf{\Lambda}$  and  $\mathbf{\Omega}$ , we substitute  $\hat{P}_{S_n}^{(j)*}$  and  $\hat{P}_R^{(k)*}$  into (21), yielding the following

$$\begin{aligned}
& e^{\hat{P}_{S_n}^{(j)2}} \underbrace{\left( \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} (\mathcal{L} + \mu) \sigma_{D_n}^{(k)2} |h_{S_n}^{(j)}|^2 \right)}_{a_1} \\
& + e^{\hat{P}_{S_n}^{(j)}} \underbrace{\sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} (\mathcal{L} + \mu) \left( |g_{RD_n}^{(k)}|^2 e^{\hat{P}_R^{(k)}} + \sigma_{D_n}^{(k)2} \right) \left( \sum_{l=1, l \neq n}^N e^{\hat{P}_{S_l}^{(j)}} |h_{S_l}^{(j)}|^2 + \sigma_R^{(j)2} \right)}_{b_1} \\
& - \underbrace{\left( \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \frac{\alpha_n^{(j,k)}}{2 \ln(2)} + \nu_n^{(j,k)} \right) \left( |g_{RD_n}^{(k)}|^2 e^{\hat{P}_R^{(k)}} + \sigma_{D_n}^{(k)2} \right) \left( \sum_{l=1, l \neq n}^N e^{\hat{P}_{S_l}^{(j)}} |h_{S_l}^{(j)}|^2 + \sigma_R^{(j)2} \right)}_{c_1} = 0; \\
\hat{P}_{S_n}^{(j)*}(u+1) &= \ln \left[ \frac{-b_1}{2a_1} + \sqrt{\left( \frac{b_1}{2a_1} \right)^2 + \frac{c_1}{a_1}} \right]^+, \tag{22}
\end{aligned}$$


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$$\begin{aligned}
& e^{\hat{P}_R^{(k)2}} \underbrace{\left( \sum_{j=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} (\mathcal{L} + \mu) \left( |g_{RD_n}^{(k)}|^2 \sum_{l=1, l \neq n}^N e^{\hat{P}_{S_l}^{(j)}} |h_{S_l}^{(j)}|^2 + |g_{RD_n}^{(k)}|^2 \sigma_R^{(j)2} \right) \right)}_{a_2} \\
& + e^{\hat{P}_R^{(k)}} \underbrace{\left( \sum_{j=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} (\mathcal{L} + \mu) \sigma_{D_n}^{(k)2} \left( \sum_{n=1}^N e^{\hat{P}_{S_n}^{(j)}} |h_{S_n}^{(j)}|^2 + \sigma_R^{(j)2} \right) \right)}_{b_2} \\
& - \underbrace{\left( \left( \sum_{j=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \frac{\alpha_n^{(j,k)}}{2 \ln(2)} + \nu_n^{(j,k)} \right) \sigma_{D_n}^{(k)2} \left( \sum_{n=1}^N e^{\hat{P}_{S_n}^{(j)}} |h_{S_n}^{(j)}|^2 + \sigma_R^{(j)2} \right) \right)}_{c_2} = 0; \\
\hat{P}_R^{(k)*}(u+1) &= \ln \left[ \frac{-b_2}{2a_2} + \sqrt{\left( \frac{b_2}{2a_2} \right)^2 + \frac{c_2}{a_2}} \right]^+, \tag{23}
\end{aligned}$$


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optimization problem:

$$\begin{aligned}
(\text{OP4}) \quad & \max_{\Lambda, \Omega} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \Phi_n^{(j,k)} + \Psi \\
\text{s.t.} \quad & (C.1) \ \& \ (C.3) - (C.5), \tag{26}
\end{aligned}$$

where  $\Phi_n^{(j,k)}$  and  $\Psi$  are explicitly given by

$$\begin{aligned}
\Phi_n^{(j,k)} &= \frac{1}{2} \left( \frac{\alpha_n^{(j,k)}}{\ln(2)} \ln \left( \hat{\Upsilon}_n^{(j,k)*} \right) + \beta_n^{(j,k)} \right) \\
& - (\mathcal{L} + \mu) \left( e^{\hat{P}_{S_n}^{(j)*}} + e^{\hat{P}_R^{(k)*}} \right); \tag{27}
\end{aligned}$$

$$\Psi = \mu P_{max} - \mathcal{L} X^C -$$

$$\sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \nu_n^{(j,k)} \left( \ln(\Upsilon_{n,min}) - \ln \left( \hat{\Upsilon}_n^{(j,k)*} \right) \right) \tag{28}$$

Note that only  $\Phi_n^{(j,k)}$  depends on subcarrier pairing and allocation variables, while  $\Psi$  remains constant for any subcarrier

combination. Furthermore, the two terms in  $\Phi_n^{(j,k)}$  demonstrate the sum rate achieved by the  $n$ -th user pair on the  $(j, k)$ -th subcarrier pair and the price paid for this allocation, respectively.

*Subcarrier Allocation:* For a given subcarrier pairing  $\Lambda$ , the optimization problem (26) becomes

$$\begin{aligned}
(\text{OP5}) \quad & \max_{\Omega} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Omega_n^{(j,k)} \Phi_n^{(j,k)} + \Psi \\
\text{s.t.} \quad & (C.1) \ \& \ (C.5), \tag{29}
\end{aligned}$$

Straightforwardly, the optimal subcarrier allocation is the one that maximizes  $\Phi_n^{(j,k)}$  for the  $n$ -th user on the  $(j, k)$ -th subcarrier pair and thus, it is given by

$$\Omega_n^{(j,k)*} = \begin{cases} 1, & \text{for } n = \arg \max_n \Phi_n^{(j,k)}, \forall j, k; \\ 0, & \text{otherwise,} \end{cases} \tag{30}$$

*Subcarrier Pairing:* To derive the optimal subcarrier allocation  $\Lambda^*$ , we substitute (30) into (26), yielding the following



problem:

$$\begin{aligned}
 (\text{OP6}) \quad & \max_{\Lambda} \quad \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Phi_n^{(j,k)*} + \Psi \\
 \text{s.t.} \quad & (C.1), (C.3) \text{ \& } (C.4), \quad (31)
 \end{aligned}$$

where  $\Phi_n^{(j,k)*} = \max_n \Phi_n^{(j,k)}$ ,  $\forall n, j, k$ . The problem (OP6) is solved by using Hungarian method [29].

### B. Master Problem: Update of Lagrangian Multipliers and the Price $\mathcal{L}$ :

The solution of the inner optimization problem in (21) is given by (24), (25), (30), and (31), hence, the dual problem (21) is differentiable. By applying the subgradient method [30], the dual variables  $\mu$  and  $\nu$  can be updated as shown in (32) and (33), where  $\varepsilon_\mu$  and  $\varepsilon_\nu$  are positive step sizes, and  $[\cdot]^+ = \max\{0, \cdot\}$ .

The optimal resource allocation  $(\hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \Lambda^*(u), \Omega^*(u))$  is obtained through iterative procedures of (22), (23), and (30)-(33) for fixed coefficients  $\alpha$  and  $\beta$ . The lower bound performance  $\mathcal{U}_{LB}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \Lambda, \Omega, \alpha, \beta)$  depends on the coefficients  $\alpha$  and  $\beta$  and thus we can improve the lower bound performance by carefully choosing the values of these two coefficients. We provide a theorem for updating  $\alpha$  and  $\beta$ .

*Theorem 1:* For  $\hat{\Upsilon}_n^{(j,k)}(u+1) = \hat{\Upsilon}_n^{(j,k)}(\hat{P}_{S_n}^{(j)*}(u), \hat{P}_R^{(k)*}(u))$ , if the coefficients  $\alpha_n^{(j,k)}(u)$  and  $\beta_n^{(j,k)}(u)$  are updated as follows:

$$\alpha_n^{(j,k)}(u+1) = \hat{\Upsilon}_n^{(j,k)}(u) \times \left(1 + \hat{\Upsilon}_n^{(j,k)}(u)\right)^{-1}; \quad (34)$$

$$\begin{aligned}
 \beta_n^{(j,k)}(u+1) = & \\
 \log_2 \left(1 + \hat{\Upsilon}_n^{(j,k)}(u)\right) - \alpha_n^{(j,k)}(u+1) \log_2 \left(\hat{\Upsilon}_n^{(j,k)}(u)\right), & \quad (35)
 \end{aligned}$$

then the lower bound performance  $\mathcal{U}_{LB}$  obtained in the  $u$ -th iteration, is monotonically increased with each iteration until convergence.

*Proof:* The proof is provided in Appendix A ■

### C. Network Energy Efficiency: Optimal Penalty/Price $\mathcal{L}^*$

As can be seen in (9), there is a relation between the achievable sum rate and the total power consumption in the network. By adjusting the price  $\mathcal{L}$ , the trade-off between the sum rate and the EE can be demonstrated. Our goal is to maximize the EE of the network under the desired user's QoS requirements. Hence, it raise the following question: How can we enhance the EE of the network while satisfying the user's QoS requirements; and achieve the maximum EE through resource allocation. To answer these questions, we first define EE metric of the relay network as below and later we provide the proof for the optimal penalty  $\mathcal{L}^*$  that can achieve the maximum EE with user's QoS requirements.

*Definition 1:* The EE of the network is defined as the ratio of the achievable sum rate to the total power dissipation, given

by using (7) and (8) as follows<sup>3</sup>:

$$\begin{aligned}
 \eta_{EE} = & \frac{\frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \log_2 \left(1 + \hat{\Upsilon}_n^{(j,k)}\right)}{\sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left(e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}}\right) + X^C} \\
 \triangleq & \frac{\mathcal{R}_T(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \Lambda, \Omega)}{P_T(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \Lambda, \Omega)}, \quad (36)
 \end{aligned}$$

*Theorem 2:* Let  $(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*, \Omega^*, \Lambda^*)$  be the optimal resource allocation of the optimization problem (OP3) for the penalty  $\mathcal{L}^*$ . If this optimal resource allocation policy satisfies the following balance equation<sup>4</sup>:

$$\mathcal{R}_T(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*, \Omega^*, \Lambda^*) - \mathcal{L}^* P_T(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*, \Omega^*, \Lambda^*) = 0, \quad (37)$$

then  $\mathcal{L}^*$  will be optimal price/penalty.

*Proof:* The proof is provided in Appendix B. ■

*Theorem 3:* If the penalty factor is updated at the  $(l+1)$ -th iteration as

$$\begin{aligned}
 \mathcal{L}^*(l+1) = & \mathcal{R}_T(\hat{\mathbf{P}}^*(l), \hat{\mathbf{P}}_R^*(l), \Lambda^*(l), \Omega^*(l)) \\
 & \times \left[ P_T(\hat{\mathbf{P}}^*(l), \hat{\mathbf{P}}_R^*(l), \Lambda^*(l), \Omega^*(l)) \right]^{-1}, \quad (38)
 \end{aligned}$$

for the local maximizer of (OP1) for the penalty  $\mathcal{L}(l)$  at the  $l$ -th iteration, then  $\mathcal{L}$  is monotonically increasing with respect to  $l$ .

*Proof:* The proof is provided in Appendix C. ■

The proposed iterative EE maximization (EEM) algorithm for resource allocation is summarized in Algorithm 1. We first set the maximum number of iteration counter for the outer and inner loop  $I_{max1}$  and  $I_{max2}$  with iteration counter  $l = 0$  and  $u = 0$ , respectively, and initialize the penalty factor with  $\mathcal{L}(l) = 0.001$ , followed by step sizes  $\epsilon_\mu$  and  $\epsilon_\nu$ , and the Lagrangian multipliers  $\mu(u)$  and  $\nu(u)$ , respectively. By using (22) and (23), we iteratively update  $\hat{\mathbf{P}}$  and  $\hat{\mathbf{P}}_R$ , followed by update of the Lagrangian multipliers  $\mu$  and  $\nu$  using (32) and (33). Next, the coefficients  $\alpha_n^{(j,k)}$  and  $\beta_n^{(j,k)}$  are updated using (34) and (35) with obtained optimal power allocation  $\hat{\mathbf{P}}$  and  $\hat{\mathbf{P}}_R$  and this process is repeated until convergence. In the next step, the subcarrier pairing and allocation  $\Lambda$  and  $\Omega$  are updated using (30) and (31). The above procedure is repeated until convergence or  $u > I_{max2}$ . In the outer loop, we update the penalty factor  $\mathcal{L}(l+1)$  using (38) and repeat this process until convergence or  $l > I_{max1}$ . Because of the local optimality of the proposed  $\mathcal{L}$ -price algorithm, the optimal price obtained in this algorithm can only guarantee that the locally optimum resource allocation in (OP1) with respect to  $\mathcal{L}^*$  is a local maximizer of the EE, i.e., close-to-optimal of the EE, formula in (36). In fact, the EE formula is a non-concave function in terms of  $\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \Lambda$ , and  $\Omega$ , and it is in general very difficult to

<sup>3</sup>Since the function  $\eta_{EE}(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R, \Lambda, \Omega)$  is non-concave, the optimality here is defined in a locally optimal sense, i.e. close-to-optimal.

<sup>4</sup>Since the problem (OP1) is non-convex, the optimal resource allocation here is referred to as a local maximizer, i.e., close-to-optimal.

$$\mu(u+1) = \left[ \mu(u) + \varepsilon_\mu(u) \left( \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left( e^{\hat{P}_{S_n}^{(j)}} + e^{\hat{P}_R^{(k)}} \right) - P_{max} \right) \right]^+; \quad (32)$$

$$\nu_n^{(j,k)}(u+1) = \left[ \nu_n^{(j,k)}(u) + \varepsilon_\nu(u) \left( \ln(\Upsilon_{n,min}) - \ln(\hat{\Upsilon}_n^{(j,k)*}) \right) \right]^+, \quad (33)$$

find the optimal price that can achieve the globally maximum EE.

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**Algorithm 1** An Iterative EEM Algorithm

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Set the maximum number of iterations  $I_{max1}$ ;  
Initialize the iteration counter  $l = 0$  and network penalty  $\mathcal{L}(l) = 0.001$ .  
**repeat** (Outer Loop)  
Set the maximum number of iterations  $I_{max2}$  and the step sizes  $\epsilon_\mu$ , and  $\epsilon_\nu$ ;  
Initialize  $\alpha$  and  $\beta$  and counter  $u = 0$ ;  
Initialize  $\hat{\mathbf{P}}(u)$ ,  $\hat{\mathbf{P}}_R(u)$ ,  $\mu(u)$  and  $\nu(u)$ ;  
**repeat** (Inner Loop)  
**repeat** (Solving problem (OP3))  
**repeat**  
Update  $\hat{\mathbf{P}}$  and  $\hat{\mathbf{P}}_R$  using (22) and (23);  
Update  $\mu$  and  $\nu$  using (32) and (33);  
**until** convergence to the optimal solution  $(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*)$ ;  
Update  $\alpha$  and  $\beta$  using (34)-(35);  
 $\hat{\mathbf{P}} \leftarrow \hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R \leftarrow \hat{\mathbf{P}}_R^*$ ;  
**until** convergence;  
Update  $\Omega$  and  $\Lambda$  using (30) and (31);  
**until** convergence to the optimal solution, i.e.,  $(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*, \Omega^*, \Lambda^*)$ ;  
Set  $\hat{\mathbf{P}}(u+1) \leftarrow \hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R(u+1) \leftarrow \hat{\mathbf{P}}_R^*$ ,  
 $\Omega(u+1) \leftarrow \Omega^*, \Lambda(u+1) \leftarrow \Lambda^*$  and  
 $u \leftarrow u+1$ ;  
**until** convergence or  $u > I_{max2}$ ;  
Update  $\mathcal{L}(l+1)$  using (38) and  $l \leftarrow l+1$ ;  
**until** convergence or  $l > I_{max1}$ .

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## V. SUBOPTIMAL EE RESOURCE ALLOCATION ALGORITHM

The computational complexity of the algorithm proposed in the previous section increases with the increasing of  $N$  and  $N_{sc}$ . In this section, we propose a low-complexity suboptimal algorithm whose performance is close to that of the EEM algorithm. A step-wise procedure of the suboptimal algorithm for solving the problem (OP3) is described as follows:

1) *Subcarrier Allocation for Fixed Power Allocation*: Firstly, we equally distribute the available transmit power among all the users over all the subcarriers as

$$\hat{P}_{S_n}^{(j)} = \hat{P}_R^{(k)} = \frac{P_{max}}{(N+1) \times N_{sc}}, \quad \forall n, j; \quad (39)$$

Next, we define an  $N \times (N_{sc} \times N_{sc})$  matrix where each element is indicating the SINR for the  $n$ -th user on the  $(j, k)$ -th subcarrier pairing. Then, we select the  $n$ -th user pair according to the following criteria:

$$\Omega_n^{(j,k)*} = \begin{cases} 1, & \text{for } n = \arg \max_n \text{SINR}_n^{(j,k)}; \\ 0, & \text{otherwise,} \end{cases} \quad (40)$$

2) *Subcarrier Pairing for Fixed Power Allocation*: In this step, we arrange the source-to-relay (SR) and the relay-to-destination (RD) subcarriers in the ascending order according to their respective channel gains. Then, we pair the corresponding subcarriers with each other in sequence, i.e. in a best-to-best and worst-to-worst fashion. Finally,  $N_{sc} \times N_{sc}$  matrix can be obtained as follows:

$$\Lambda^{j,k*} = \begin{cases} 1, & \text{for } j^{th} \text{ subcarrier paired with } k^{th} \text{ subcarrier;} \\ 0, & \text{otherwise,} \end{cases} \quad (41)$$

3) *Power Allocation for Obtained Subcarrier Allocation and Pairing*: The last step is to find the optimal power allocation  $(\hat{\mathbf{P}}, \hat{\mathbf{P}}_R)$  and this can be done by solving the optimization problem (OP3) for the obtained  $\Lambda^*$  and  $\Omega^*$  in the step 1 and 2, respectively.

## VI. PERFORMANCE ANALYSIS

### A. Two-User Cases Analysis

To get more insight of the EEM resource allocation policy, we study the two-user scenario in two different noise regimes namely: 1) relay noise-dominated regime and 2) destination noise-dominated regime.

1) *Relay Noise-Dominated Regime*: In this regime, the relay is assumed to operate at very low SNR region, thus the noise produced at the relay node is significantly high as compared to that of the interference-plus-destination noise, i.e.,  $\rho_R^{(k)^2} P_2^{(j)} |g_{RD1}^{(k)} h_{S2}^{(j)}|^2 + \sigma_{D1}^{(k)^2} \ll \rho_R^{(k)^2} |g_{RD1}^{(k)}|^2 \sigma_R^{(j)^2}$  and  $\rho_R^{(k)^2} P_1^{(j)} |g_{RD2}^{(k)} h_{S1}^{(j)}|^2 + \sigma_{D2}^{(k)^2} \ll \rho_R^{(k)^2} |g_{RD2}^{(k)}|^2 \sigma_R^{(j)^2}$ . For the optimal subcarrier pairing  $\Lambda^*$ , and subcarrier allocation  $\Omega^*$ , the power allocation policy for users works as water-filling and it is described as

$$P_{S_n}^{(j)*} = \left[ \frac{1/(2 \ln 2)}{\left( \mathcal{L} + \mu^* - \nu_n^{(j,k)} \frac{|h_{S_n}^{(j)}|^2}{\sigma_R^{(j)^2}} \right)} - \frac{\sigma_R^{(j)^2}}{|h_{S_n}^{(j)}|^2} \right]^+, \quad (42)$$

for  $n = 1, 2$ .

2) *Destination Noise-Dominated Regime*: The noise produced at the destination nodes is significantly higher than the interference-plus-relay noise and the relay noise, i.e.  $\rho_R^{(k)^2} P_2^{(j)} |g_{RD1}^{(k)} h_{S2}^{(j)}|^2 + \rho_R^{(k)^2} |g_{RD1}^{(k)}|^2 \sigma_R^{(j)^2} \ll \sigma_D^2$  and  $\rho_R^{(k)^2} P_1^{(j)} |g_{RD2}^{(k)} h_{S1}^{(j)}|^2 + \rho_R^{(k)^2} |g_{RD2}^{(k)}|^2 \sigma_R^{(j)^2} \ll \sigma_D^2$  where

$\sigma_D^2 = \sigma_{D_2}^{(k)^2} = \sigma_{D_2}^{(k)^2}$ . Under this regime, the ratio of  $\hat{P}_{S_1}^{(j)*}$  to  $\hat{P}_{S_2}^{(j)*}$  can be written as

$$\frac{\hat{P}_{S_1}^{(j)*}}{\hat{P}_{S_2}^{(j)*}} = \frac{\left( \frac{\Lambda^{j,k} \Omega_1^{(j,k)} \alpha_1^{(j,k)}}{2 \ln(2)} + \nu_1^{(j,k)} \right) \left[ 1 + \frac{|h_{S_2}^{(j)}|^2}{|h_{S_1}^{(j)}|^2} \right]}{\left( \frac{\Lambda^{j,k} \Omega_2^{(j,k)} \alpha_2^{(j,k)}}{2 \ln(2)} + \nu_2^{(j,k)} \right) \left[ 1 + \frac{|h_{S_1}^{(j)}|^2}{|h_{S_2}^{(j)}|^2} \right]}. \quad (43)$$

The result in (43) reveals a channel-reversal power allocation strategy. The user with a lower source-to-relay channel gain is assigned higher transmit power.

### B. Complexity Analysis

In this section, we perform an exhaustive computational complexity analysis to get a better insight into the complexity reduced by the proposed EEM and suboptimal algorithms compared to the complexity of the exhaustive search, which gives the globally optimal solution. It is assumed that the network price  $\mathcal{L}$  converges in  $U$  iterations.

Firstly, the complexity of EEM algorithm is analyzed. To find the optimal power allocation solution of (OP3), we require to solve  $NN_{sc}^2$  subproblems due to  $N$  number of user pairs operating on  $N_{sc}$  subcarriers in each hop. Since the optimal resource allocation  $(\hat{\mathbf{P}}^*, \hat{\mathbf{P}}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)$  is determined under the total power constraint (C.1) and the QoS constraint (C.2) and thus the complexity resulted from these two constraints is  $\mathcal{O}(V^3 + 2)$ , where  $V$  denotes the maximum power level for each source and the relay nodes on each subcarrier. Furthermore, the complexity added due to maximization problem (29) is  $\mathcal{O}(N)$ . Moreover, the Hungarian method is used to determine the optimal subcarrier pairing in (31) with a complexity of  $\mathcal{O}(N_{sc}^3)$ . The complexity for updating a dual variable is  $\mathcal{O}((2N)^\varpi)$  (for example,  $\varpi = 2$  if the ellipsoid method is used [31]). Thus, the total complexity for updating dual variables is  $\mathcal{O}(2(2N)^\varpi)$ . Let us suppose if the dual objective function  $g(\mu, \nu)$  converges in  $W$  iterations, then total complexity for the EEM algorithm is  $\mathcal{O}(2UWN_{sc}^2(2N)^\varpi(N(V^3 + 3) + N_{sc}))$ . In addition, the complexity of EEM algorithm under equal subcarrier power allocation (ESPA) can be given as  $\mathcal{O}(2UWN_{sc}^2(2N)^\varpi(N(V^3 + 3) + N_{sc}))$ .

In case of the suboptimal algorithm, the implementation of the subcarrier allocation in step 1 requires a complexity of  $\mathcal{O}(NN_{sc})$ , whereas the subcarrier pairing in step 2 imposes a complexity of  $\mathcal{O}(2N_{sc})$ . However, the power allocation adds a complexity of  $\mathcal{O}(V^3 + 2)$  and  $\mathcal{O}(2(2N)^\varpi)$  for  $NN_{sc}^2$  subproblems in a similar way to the EEM algorithm, respectively. If the dual objective function  $g(\mu, \nu)$  converges in  $W'$  iterations (without loss of generality let  $W' = W$ ), then the total complexity for the suboptimal algorithm becomes  $\mathcal{O}(2UWN_{sc}(2N)^\varpi(N + 2 + NN_{sc}(V^3 + 2)))$ . The complexity of the exhaustive search is given by  $\mathcal{O}(2UW(2N)^\varpi N^{N_{sc}!}(V^3 + 2))$ .

## VII. EXTENSION OF DESIGN FRAMEWORK

It is noteworthy that the design framework can be easily extended to accommodate a more general scenarios in multiuser AF relay network as follows: 1) multiuser relay network with an eavesdropper and 2) a two-phase multiuser full-duplex multi-relay network. We discuss the extensibility of our design framework as well as joint subcarrier pairing, subcarrier and power allocation methodology to these two scenarios in the following.

### A. Multiuser Relay Network with an Eavesdropper

Our design framework can be extended to accommodate this scenario by modifying the network utility function. The signal received from source node  $S_n$  at the eavesdropper node on the  $j$ -th subcarrier can be written as

$$y_{S_n E}^{(j)} = h_{S_n E}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)} + n_E^{(j)}, \quad (44)$$

where  $h_{S_n E}^{(j)}$  denotes the channel fading coefficient from the  $n$ -th source node to the eavesdropper on the  $j$ -th subcarrier and  $n_E^{(j)} \sim \mathcal{CN}(0, \sigma_E^2)$ ,  $\forall j$ , is the AWGN at the eavesdropper with power  $\sigma_E^2$ . Then the corresponding SNR is

$$\gamma_{S_n E}^{(j)} = \frac{P_{S_n}^{(j)} |h_{S_n E}^{(j)}|^2}{\sigma_E^2}, \quad (45)$$

The capacity of the illegal link for the  $n$ -th source node is written as

$$\mathcal{R}_{S_n E} = \frac{1}{2} \log_2 \left( 1 + \gamma_{S_n E}^{(j)} \right), \quad (46)$$

From (6) and (46), the secrecy rate of the network can be expressed as

$$\begin{aligned} \mathcal{R}_S(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) &= \sum_{n=1}^N [\mathcal{R}_n - \mathcal{R}_{S_n E}]^\dagger; \\ &= \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{j,k} \Omega_n^{(j,k)} \left[ \log_2 \left( \frac{1 + \Upsilon_n^{(j,k)}}{1 + \gamma_{S_n E}^{(j)}} \right) \right]^\dagger, \end{aligned} \quad (47)$$

Then the network utility function can be formulated as

$$\begin{aligned} \mathcal{U}_S(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) &= \\ &\mathcal{R}_S(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) - \mathcal{L} P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}), \end{aligned} \quad (48)$$

Using (48), the new optimization problem can be formulated and solved in a similar way as the problem (OP1). This model can be easily generalized to the scenario of multiple eavesdroppers in wireless networks.

### B. Multiuser Relay Network with Full-Duplex Multi-Relay

Full-duplex (FD) relaying has potential to double the SE by transmitting and receiving simultaneously in the same frequency band. However, the self-interference (SI) caused by the signal leakage dominates the performance of FD relaying system. Thanks to the advancement in interference cancellation techniques and transmit/receive antenna isolation [27] which enabled the FD relaying to combat with SI, and thereby leading to FD relaying well deserved attention from

both industry and academic [27]. The design framework can be extended to multiuser FD multi-relay network where  $M$  relay nodes  $R_m$ ,  $m \in \{1, \dots, M\}$  operate in FD mode and are equipped with a single antenna for enabling simultaneous transmission and reception<sup>5</sup>.

Let  $P_{S_n}^{(j)}$ ,  $P_{R_m}^{(k)}$  denote the power transmitted by source  $S_n$  and relay  $R_m$  on subcarriers  $j$  and  $k$ , respectively. At the time instant  $t$ , the  $N$  source nodes concurrently transmit signals to all the relay nodes. Thus, the received signal at the  $m$ -th relay node on subcarrier  $j$  can be given by

$$y_{R_m}^{(j)}(t) = \sum_{n=1}^N h_{S_n, R_m}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)}(t) + h_{R_m}^{(j)} x_{R_m}^{(k)}(t) + n_{R_m}^{(j)}(t), \quad (49)$$

where  $h_{S_n, R_m}^{(j)}$  and  $x_{S_n}^{(j)}$  represent the channel coefficient from the  $n$ -th source node to the  $m$ -th relay node and the transmit signal by the  $n$ -th source node on subcarrier  $j$  at time instant  $t$ , respectively, with  $\mathbb{E} \left[ x_{S_n}^{(j)} \right]^2 = 1$ ,  $n_{R_m}^{(j)}(t)$  indicates the AWGN at the relay node on subcarrier  $j$  with zero mean and variance  $\mathbb{E} \left[ n_{R_m}^{(j)} \right]^2 = \sigma_{R_m}^{(j)^2}$ .  $h_{R_m}^{(j)}$  denotes the SI channel on subcarrier  $j$  at the  $m$ -th relay node.  $x_{R_m}^{(k)}(t)$  is the signal transmitted by the  $m$ -th relay node at the time instant  $t$  on subcarrier  $k$  and is expressed as

$$x_{R_m}^{(k)}(t) = \rho_{R_m}^{(k)} y_{R_m}^{(j)}(t-1), \quad (50)$$

By substituting (49) into (50) and taking into account the relay transmit power  $P_{R_m}^{(k)}$ , the normalized amplification factor  $\rho_{R_m}^{(k)}$  can be written as

$$\rho_{R_m}^{(k)} = \sqrt{\frac{P_{R_m}^{(k)}}{\sum_{n=1}^N P_{S_n}^{(j)} \left| h_{S_n, R_m}^{(j)} \right|^2 + P_{R_m}^{(k)} \left| h_{R_m}^{(j)} \right|^2 + \sigma_{R_m}^{(j)^2}}}, \quad (51)$$

The received signal at the destination node  $D_n$  from the relay node  $R_m$  on the  $k$ -th subcarrier can be expressed as in (52), as shown on the top of the next page. From (52), the equivalent SINR of the virtual channel between source  $S_n$  and destination  $D_n$  can be written as in (53), as shown on the top of the next page. Similar to (7) and (8), the total sum rate and the power dissipation in the network can be mathematically posed as

$$\mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{A}, \mathbf{\Omega}) = \sum_{n=1}^N \sum_{m=1}^M \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{(j,k)} \Omega_{n,m}^{(j,k)} \log_2 \left( 1 + \Upsilon_{n,m}^{(j,k)} \right), \quad (54)$$

$$P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{A}, \mathbf{\Omega}) = \sum_{n=1}^N \sum_{m=1}^M \sum_{j=1}^{N_{sc}} \sum_{k=1}^{N_{sc}} \Lambda^{(j,k)} \Omega_{n,m}^{(j,k)} \left( P_{S_n}^{(j)} + P_{R_m}^{(k)} \right) + X^C, \quad (55)$$

where  $\mathbf{P}_R = \{P_{R_m}^{(k)}\}$ ,  $\forall m, k$  and  $\mathbf{\Omega} = \{\Omega_{n,m}^{(j,k)}\}$ ,  $\forall n, m, j, k$ . If the user pair  $n$  is communicating with the assistance of the relay  $m$  over subcarrier pair  $(j, k)$ ,  $\Omega_{n,m}^{(j,k)}$  is one, otherwise it

is zero. Using (54) and (55), the network utility function can be mathematically written as

$$\mathcal{U}(\mathbf{P}, \mathbf{P}_R, \mathbf{A}, \mathbf{\Omega}) = \mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{A}, \mathbf{\Omega}) - \mathcal{L} P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{A}, \mathbf{\Omega}), \quad (56)$$

Using (56), the optimization problem for multiuser full-duplex multi-relay network can be formulated and solved in a similar way as the problem (OP1), but with modification of the constraints (C.5) of the problem (OP1) as follows:

$$\begin{aligned} \sum_{m=1}^M \Omega_{m,n}^{(i,j)} &= 1, & \forall n, (j, k); \\ \sum_{n=1}^N \Omega_{m,n}^{(i,j)} &= 1, & \forall m, (j, k), \end{aligned}$$

*Remark:* Multi-relay has the advantage of increasing the diversity gain and flexibility of the network compared with single relay. In addition, multi-relay cooperation can solve the over load problem of single relay. However, the multi-relay brings up a lot of new challenges. It is evident from [3] that relay stations consume up to fifty-five percent of the entire power in a typical wireless network. In multi-relay cooperative networks, if all the relay stations are active, the static power consumption of relay stations is considerably high and this could be a significant portion of the whole power consumption. Besides, the system complexity increases with the increasing the number of relays in the networks. For example: the complexity of the EEM algorithm under FD multi-relay is  $\mathcal{O}(2UWN_{sc}^2(2N)^\varpi(MN(V^3+3)+N_{sc}))$ . In contrast, single relay is of importance for practical implementation because it not only reduces the unnecessary static power consumption and the system complexity but also improves the system spectral efficiency (SE) while maintaining its diversity order.

## VIII. NUMERICAL RESULTS

In this section, we present the simulation results to demonstrate the effectiveness of the proposed resource allocation algorithms. The considered multiuser relay network is composed of  $N$  user pairs and a single relay node. For simulation, we consider both the Rayleigh fading and the log-normal shadowing effects which are given by  $\mathcal{CN}(0, 1)$  and  $\ln \mathcal{N}(0, 8 \text{ dB})$ , respectively. A path loss model in the Third-Generation Partnership Project (3GPP), described as  $131.1 + 42.8 \times \log_{10}(d) \text{ dB}$  ( $d$ : distance in km) [33] is taken into consideration. The QoS requirement for each user per subcarrier pair is given as  $\Upsilon_{m,min} \geq -20 \text{ dB}$ . The maximum transmit power is set as  $P_{max} = 25 \text{ dBm}$ . The maximum number of iterations and the convergence tolerance value in the proposed algorithms are 10 and  $10^{-5}$ , respectively, while circuit and processing power dissipation at each node is assumed to be 10 dBm. The subcarrier spacing and the thermal noise density are considered as 12 kHz and  $-174 \text{ dBm/Hz}$ , respectively.  $d_{SR}$  and  $d_{RD}$  denote the distance from all source nodes to the relay and from the relay to all destination nodes, respectively. The performance of the proposed algorithms with

<sup>5</sup>Note that FD radio prototypes equipped with a circulator can transmit and receive signals simultaneously with a single antenna [28].



$$\begin{aligned}
y_{R_m D_n}^{(k)}(t) = & \underbrace{g_{R_m D_n}^{(k)} \rho_{R_m}^{(k)} h_{S_n R_m}^{(j)} \sqrt{P_{S_n}^{(j)}} x_{S_n}^{(j)}(t-1)}_{\text{Desired Signal}} + \underbrace{g_{R_m D_n}^{(k)} \rho_{R_m}^{(k)} \sum_{l=1, l \neq n}^N h_{S_l R_m}^{(j)} \sqrt{P_{S_l}^{(j)}} x_{S_l}^{(j)}(t-1)}_{\text{MultiUser Interference}} \\
& + \underbrace{g_{R_m D_n}^{(k)} \rho_{R_m}^{(k)} h_{R_m}^{(j)} x_{R_m}^{(k)}(t-1)}_{\text{Self Interference}} + \underbrace{g_{R_m D_n}^{(k)} \rho_{R_m}^{(k)} n_{R_m}^{(j)}(t-1) + n_{D_n}^{(k)}(t)}_{\text{Noise}}, \tag{52}
\end{aligned}$$

$$\Upsilon_{n,m}^{(j,k)} = \frac{\rho_{R_m}^{(k)} P_{S_n}^{(j)} |h_{S_n R_m}^{(j)} g_{R_m D_n}^{(k)}|^2}{\rho_{R_m}^{(k)} |g_{R_m D_n}^{(k)}|^2 \sum_{l=1, l \neq n}^N P_{S_l}^{(j)} |h_{S_l R_m}^{(j)}|^2 + \rho_{R_m}^{(k)} |g_{R_m D_n}^{(k)} h_{R_m}^{(j)}|^2 + \rho_{R_m}^{(k)} |g_{R_m D_n}^{(k)}|^2 \sigma_{R_m}^{(j)^2} + \sigma_{D_n}^{(k)^2}}, \tag{53}$$

FD relaying depend on the effective self-interference cancellation methods [27], and thus the self-interference cancellation amount of FD is set to 60 dB. The proposed algorithms are compared with the following algorithms:

- Exhaustive search (ES) algorithm: This algorithm explores over all variables for each possible solutions to give the optimal solution.
- A scheme in Ref. [20]: This is power allocation scheme for relay-assisted multiuser networks, and the performance of this work is included with QoS constraints and without beamforming for a fair comparison.
- SEM algorithm: This is conventional sum rate maximization problem, which is also a degenerated case of our proposed EEM resource allocation scheme when  $\mathcal{L} = 0$ .
- ESPA algorithm: Under this case, the available power budget is equally distributed over all the subcarriers among all the users.

We first look into the convergence behavior of the proposed algorithms and the effect of price on average EE and SE. Fig. 2(a) shows the convergence behavior of the ES and the proposed algorithms for a single channel realization, where  $N = 2$ ,  $N_{sc} = \{2, 4\}$ ,  $P_{max} = 25$  dBm and  $d_{SR} = d_{RD} = 200$  m. As can be seen that the EE performance of the ES and the proposed algorithms monotonically increases with each iteration initially, however, it converges in less than five iterations. Fig. 2(b) presents the effect of different price  $\mathcal{L}$  on the average EE and SE performance of the proposed EEM algorithm when  $N = 2$ ,  $N_{sc} = 4$ ,  $P_{max} = 10$  dBm and  $d_{SR} = d_{RD} = 200$  m. As expected, the average SE performance decreases as the price  $\mathcal{L}$  increases, whereas we achieve the maximum EE in terms of the price at  $\mathcal{L} = 0.2$ . This leads to a trade-off between the EE and SE.

Fig. 3 depicts the performance comparison of various algorithms. The parameter settings are  $N = 2$ ,  $N_{sc} = 4$ , and  $d_{SR} = d_{RD} = 200$  m. As can be observed that the average EE is significantly improved as  $P_{max}$  increases and when  $P_{max} \leq 10$  dBm, the optimal ES, the EEM, suboptimal and SEM algorithms exhibit approximately identical average EE or SE performance due to limited power budget. However, when  $P_{max} > 10$  dBm, the average EE performance of the ES, EEM and suboptimal algorithms become constant, whereas that of the SEM algorithm quickly declines as  $P_{max}$

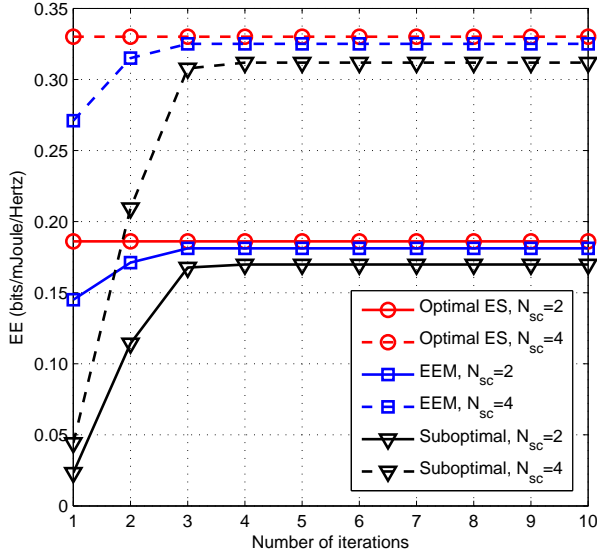
increases because each user utilizes maximum transmit power in order to enhance their sum rate. On the other side, the average SE of the proposed algorithms is slowly saturated for  $P_{max} > 10$  dBm, while the performance of the SEM algorithm is continuously improved as  $P_{max}$  increases. The average EE and SE performance of Ref. [20] with QoS constraint and ESPA is worst as compared to that of the EEM and SEM algorithms.

Fig. 4 shows the effect of the number of subcarriers  $N_{sc}$  on the average EE and SE of the proposed algorithms. The parameter settings are  $N = 2$  and  $d_{SR} = d_{RD} = 200$  m. It can be observed that the average EE can be significantly improved as  $P_{max}$  increases and when  $P_{max} \leq 10$  dBm, the EEM and suboptimal algorithms exhibit approximately identical average SE performance due to limited power budget. However, when  $P_{max} > 15$  dBm, the average EE and SE performance of both algorithms become constant. When the number of subcarriers  $N_{sc}$  increases, the average EE improves due to the frequency diversity.

The effect of number of users on the average EE and SE performance is depicted in Fig. 5 for  $N_{sc} = 6$  and  $d_{SR} = d_{RD} = 200$  m. As expected, the average EE performance of the EEM algorithm decreases when  $N$  increases due to increase in the static power dissipation. On the other hand, the average SE is improved because of the multiuser diversity. From 5, we can observed that when  $P_{max} > 10$  dBm, the average SE of the EEM algorithm improves at the cost of a degradation in the average EE.

Fig. 6 shows the average EE and SE performance comparison of FD and HD relaying networks under  $N = 2$ ,  $M = 1$ ,  $N_{sc} = 8$ , and  $d_{SR} = d_{RD} = 200$  m. It further reveals that the average EE and SE of the proposed algorithms can be rapidly enhanced as  $P_{max}$  increases, and it becomes constant when  $P_{max} \geq 10$  dBm. Moreover, the proposed algorithms with FD outperforms the existing HD, because HD requires much higher transmission power to achieve higher throughput.

Fig. 7 shows the average EE and SE performance comparison of the proposed EEM algorithm for HD relaying network in the presence and absence of an eavesdropper node with  $N = 2$ ,  $N_{sc} = 4$ , and  $d_{SR} = d_{RD} = 200$  m. It is evident that the average EE and SE of the proposed algorithms can be rapidly enhanced as  $P_{max}$  increases, and it becomes steady for



(a) EE versus Number of iterations

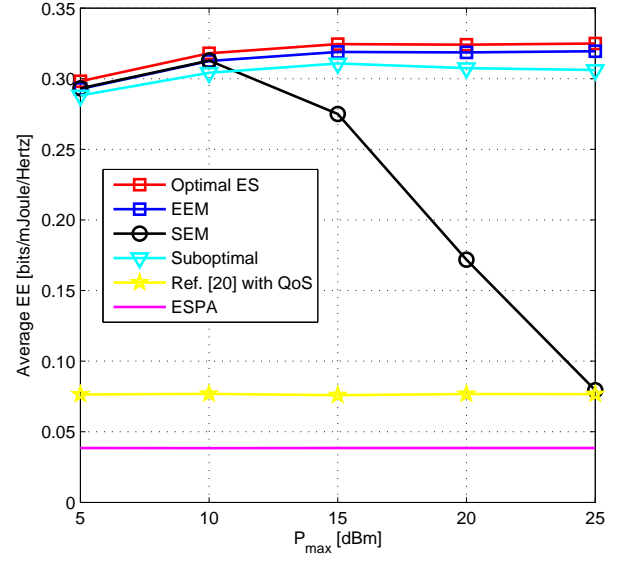
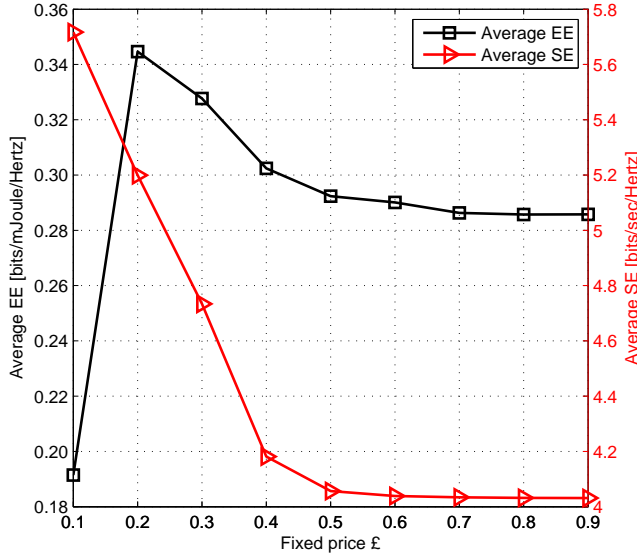
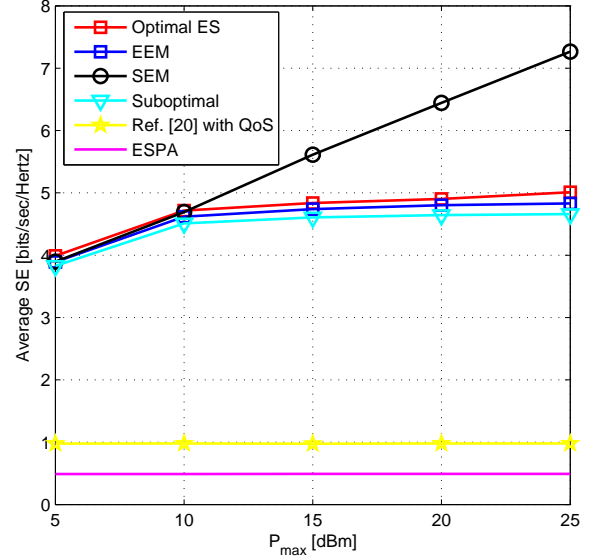
(a) Average EE versus  $P_{max}$ (b) The effect of different network prices  $\mathcal{L}$  on average EE and SE(b) Average SE versus  $P_{max}$ 

Fig. 2. Convergence behavior of the proposed algorithms and effect of different network prices on average EE and SE.

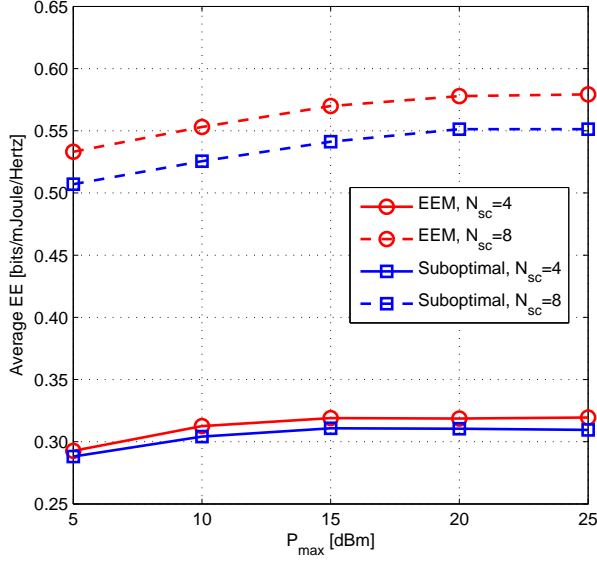
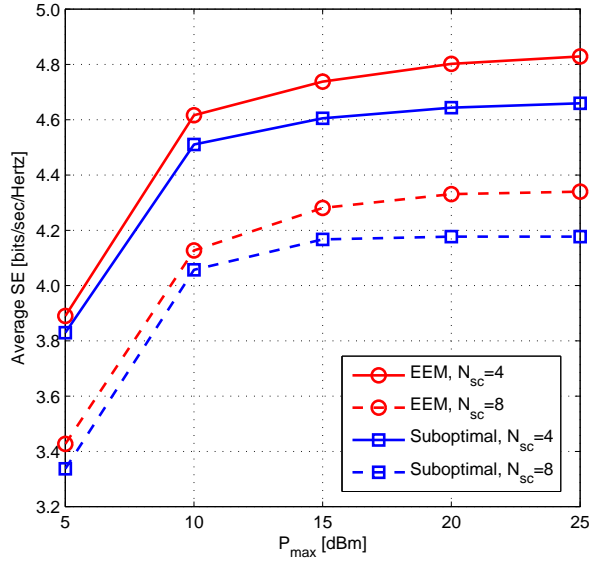
Fig. 3. Performance comparison of various algorithms with  $N = 2$ ,  $N_{sc} = 4$ , and  $d_{SR} = d_{RD} = 200$  m.

higher power regimes, i.e.,  $P_{max} \geq 15$  dBm. Moreover, the proposed algorithms without eavesdropper node outperforms the system with eavesdropper, because eavesdropper node acts as a malicious node, thereby, it fetches a packet and does not forward it to the destination nodes, hence it significantly degrades the performance of the network, especially, when compared to the system without eavesdropper node.

## IX. CONCLUDING REMARKS AND FUTURE DIRECTIONS

In this paper, we have investigated how to find best subcarrier allocation, subcarrier pairing and power allocation in relay-assisted multiuser AF relay networks in the direction of ameliorating enhance the energy utilization among users,

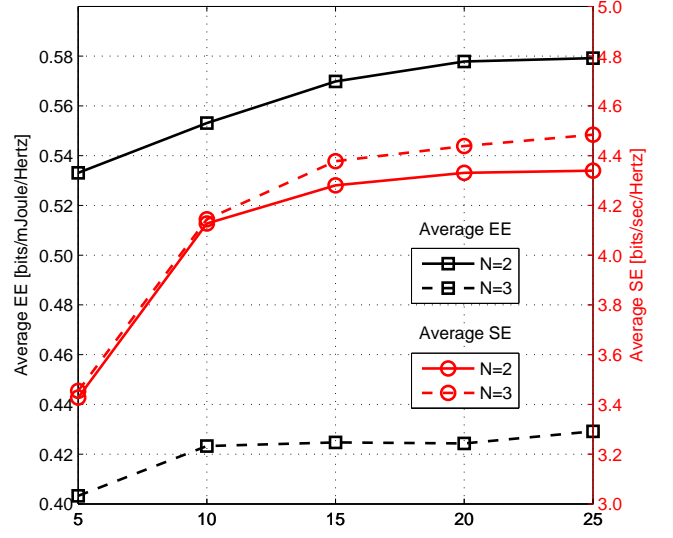
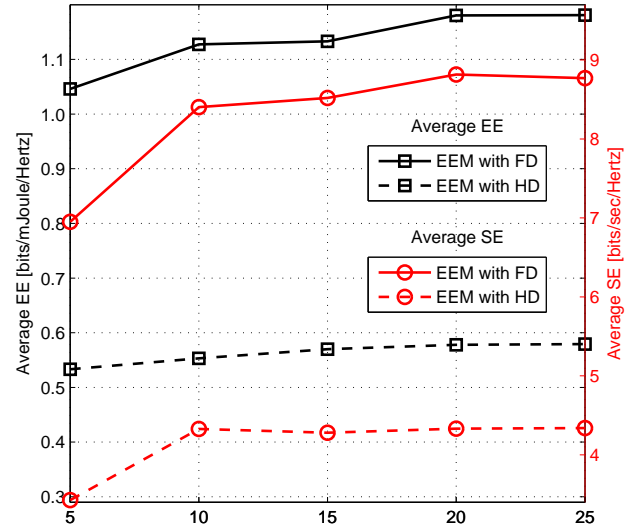
subject to a total transmit power and user QoS constraints. A network utility-based resource allocation problem was formulated. The original problem was a non-convex MINLP problem. We transformed this problem into a convex one through a series of transformations and thereby, obtained the near-optimal solution through the proposed iterative EEM algorithms. We also illustrated the relationship between  $\mathcal{L}$  and the network EE. To counterbalance the complexity, a suboptimal algorithm was also investigated showcasing comparable performance gains through simulation results. For more insights, complexity of various algorithms and two-user case for different noise regimes were analyzed. Further, the performance of the proposed algorithms were compared with

(a) Average EE versus  $P_{max}$ (b) Average SE versus  $P_{max}$ Fig. 4. Effect of number of subcarriers on the average EE and SE under  $N = 2$  and  $d_{SR} = d_{RD} = 200$  m.

that of the SEM and Ref. [20] with QoS by extensive computer simulations, and the effect of the number of subcarriers and the users were shown. The simulation results demonstrate that the performance of the proposed algorithms is superior to that of the other candidates. The inclusion of user scheduling, multi-antennas, MIMO multi-relay, relay selection, and antenna beamforming will be considered in the future works.

#### APPENDIX A PROOF OF THEOREM 1

For the coefficients  $\alpha(u)$  and  $\beta(u)$ , let  $(\hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \Lambda^*(u), \Omega^*(u))$  be the optimal solution.

Fig. 5. Effect of number of users on the average EE and SE performance of the EEM algorithm ( $N_{sc} = 6$  and  $d_{SR} = d_{RD} = 200$  m).Fig. 6. Performance of EEM algorithm with FD and HD for  $N = 2$ ,  $M = 1$ ,  $N_{sc} = 8$  and  $d_{SR} = d_{RD} = 200$  m.

Then, if we update these coefficients using (34) and (35) in accordance with (12)–(14), we get

$$\begin{aligned} & \mathcal{U}_{LB} \left( \hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \Lambda^*(u), \Omega^*(u), \alpha^*(u), \beta^*(u) \right) \\ & \leq \mathcal{U} \left( \hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \Lambda^*(u), \Omega^*(u) \right) \\ & = \mathcal{U}_{LB} \left( \hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \Lambda^*(u), \Omega^*(u), \alpha^*(u+1), \beta^*(u+1) \right); \end{aligned} \quad (\text{A.1})$$

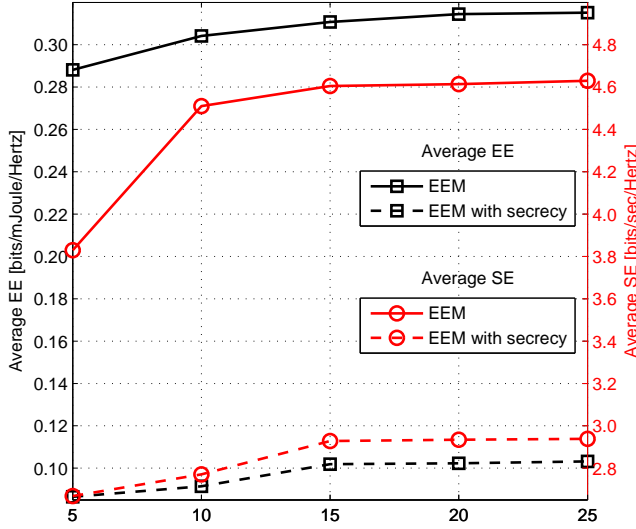


Fig. 7. Performance of EEM algorithm in the presence of an eavesdropper with  $N = 2$ ,  $N_{sc} = 4$  and  $d_{SR} = d_{RD} = 200$  m.

From optimization problem (OP3) it directly implies that

$$\begin{aligned} \mathcal{U}_{LB} \left( \hat{\mathbf{P}}^*(u), \hat{\mathbf{P}}_R^*(u), \mathbf{\Lambda}^*(u), \mathbf{\Omega}^*(u), \alpha^*(u+1), \beta^*(u+1) \right) \\ \leq \mathcal{U}_{LB} \left( \hat{\mathbf{P}}^*(u+1), \hat{\mathbf{P}}_R^*(u+1), \mathbf{\Lambda}^*(u+1), \mathbf{\Omega}^*(u+1), \right. \\ \left. \alpha^*(u+1), \beta^*(u+1) \right), \end{aligned} \quad (\text{A.2})$$

Hence, the lower bound performance  $\mathcal{U}_{LB}$  is monotonically increased with each iteration until convergence.

#### APPENDIX B PROOF OF THEOREM 2

Let  $\mathcal{Y}$  be the feasible set of the problem (OP1) and  $(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) \in \mathcal{Y}$ , then

$$\begin{aligned} \mathcal{L} &= \max_{\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}} \frac{\mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})}{P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})} = \frac{\mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}{P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}; \\ &\Rightarrow \frac{\mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}{P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)} \geq \frac{\mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})}{P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega})}, \end{aligned} \quad (\text{B.1})$$

From (B.1), we have

$$\mathcal{R}_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) - \mathcal{L} P_T(\mathbf{P}, \mathbf{P}_R, \mathbf{\Lambda}, \mathbf{\Omega}) \leq 0; \quad (\text{B.2})$$

$$\mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*) - \mathcal{L} P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*) = 0. \quad (\text{B.3})$$

As can be seen in (B.3), the local maximum is achieved by  $(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)$ . Let us suppose  $\mathcal{L}^*$  is the optimal penalty, this implies that

$$\mathcal{L}^* = \frac{\mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}{P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}, \quad (\text{B.4})$$

From (B.2) and (B.4), we get

$$\frac{\mathcal{R}_T(\mathbf{P}_j, \mathbf{P}_{R_k}, \mathbf{\Lambda}, \mathbf{\Omega})}{P_T(\mathbf{P}_j, \mathbf{P}_{R_k}, \mathbf{\Lambda}, \mathbf{\Omega})} \leq \mathcal{L}^* = \frac{\mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}{P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)}; \quad (\text{B.5})$$

$$\triangleq \mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*) - \mathcal{L}^* P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*), \quad (\text{B.6})$$

The equation (B.6) shows that when the optimal resource allocation strategies satisfy the balance equation, we get the optimal penalty  $\mathcal{L}^*$ .

#### APPENDIX C PROOF OF THEOREM 3

Let  $Q(\mathcal{L}(l)) = \mathcal{R}_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*) - \mathcal{L}^* P_T(\mathbf{P}^*, \mathbf{P}_R^*, \mathbf{\Lambda}^*, \mathbf{\Omega}^*)$  be a function in a sequence  $\mathcal{L}(l)$ , we have

$$\begin{aligned} Q(\mathcal{L}(l)) &\geq \mathcal{R}_T(\mathbf{P}^*(l-1), \mathbf{P}_R^*(l-1), \mathbf{\Lambda}^*(l-1), \mathbf{\Omega}^*(l-1)) \\ &\quad - \mathcal{L}(l) P_T(\mathbf{P}^*(l-1), \mathbf{P}_R^*(l-1), \mathbf{\Lambda}^*(l-1), \mathbf{\Omega}^*(l-1)) \end{aligned} \quad (\text{C.1})$$

where the coefficients  $\alpha(l-1)$  and  $\beta(l-1)$  are updated in accordance with  $\mathcal{L}(l)$ . From (C.1)

$$\begin{aligned} Q(\mathcal{L}(l)) &= \mathcal{R}_T(\mathbf{P}^*(l), \mathbf{P}_R^*(l), \mathbf{\Lambda}^*(l), \mathbf{\Omega}^*(l)) \\ &\quad - \mathcal{L}(l) P_T(\mathbf{P}^*(l), \mathbf{P}_R^*(l), \mathbf{\Lambda}^*(l), \mathbf{\Omega}^*(l)) \\ &= (\mathcal{L}(l+1) - \mathcal{L}(l)) P_T(\mathbf{P}^*(l), \mathbf{P}_R^*(l), \mathbf{\Lambda}^*(l), \mathbf{\Omega}^*(l)) \geq 0 \end{aligned} \quad (\text{C.2})$$

Since  $P_T(\mathbf{P}_j^*(l), \mathbf{P}_{R_k}^*(l), \mathbf{\Lambda}^*(l), \mathbf{\Omega}^*(l)) \geq 0 \Rightarrow \mathcal{L}(l+1) \geq \mathcal{L}(l)$ . Hence this theorem is proved.

#### REFERENCES

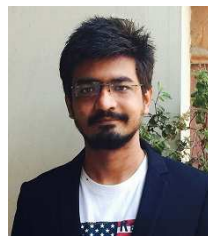
- [1] S. Schelder and V. Kuhn, "Resource allocation for multiple-access relay channels and OFDMA," *EURASIP J. on Advances in Sig. Process.*, Dec. 2013.
- [2] T. Han and N. Ansari, "On greening cellular networks via multicell cooperation," *IEEE Wireless Commun.*, vol. 20, no. 1, pp. 82-89, Feb. 2013.
- [3] Z. Hasan, H. Boostanimehr, and V. K. Bhargava, "Green cellular networks: a survey, some research issues and challenges," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 4, pp. 524-540, Nov. 2011.
- [4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037-3063, Sep. 2005.
- [5] W. Su, A. K. Sadek, and K. J. Ray Liu, "Cooperative communication protocols in wireless networks: performance analysis and optimum power allocation," *Wireless Personal Commun.*, vol. 44, no. 2, pp. 181-217, Jan. 2008.
- [6] M. R. Souryal and B. R. Vojcic, "Performance of amplify-and-forward and decode-and-forward relaying in Rayleigh fading with turbo codes," in *Proc. IEEE ICASSP*, pp. 681-684, May 2006.
- [7] N. Abughalieh, K. Steenhaut, A. Nowe, and A. Anpalagan, "Turbo codes for multi-hop wireless sensor networks with decode-and-forward mechanism," *EURASIP J. Wireless Commun. and Networking*, pp. 1-13, Nov. 2014.
- [8] K. Singh, M.-L. Ku, and J.-C. Lin, "Joint QoS-promising and EE-balancing power allocation for two-way relay networks," in *Proc. IEEE PIMRC*, pp. 1781-1785, Aug.-Sep. 2015.
- [9] O. Arnold, F. Richter, G. Fettweis, and O. Blume, "Power consumption modeling of different base station types in heterogeneous cellular networks," in *IEEE Proc. FNMS*, pp. 1-8, Jun. 2010.
- [10] K. T. Phan, T. Le-Ngoc, S. A. Vorobyov and C. Tellambura, "Power allocation in wireless multi-user relay networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2535-2545, May 2009.
- [11] Y. Liu and A. P. Petropulu, "QoS guarantees in AF relay networks with multiple source-destination pairs in the presence of imperfect CSI," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4225-4335, Sep. 2013.
- [12] A. K. Sadek, Z. Han, and K. J. R. Liu, "Distributed relay-assignment protocols for coverage expansion in cooperative wireless networks," *IEEE Trans. Mobile Comput.*, vol. 9, no. 4, pp. 505-515, Apr. 2010.
- [13] Y. Dai and X. Dong, "Power allocation for multi-pair massive MIMO two-way AF relaying with linear processing," *IEEE Wireless Commun. Lett.*, vol. 15, no. 9, pp. 5932-5946, Sep. 2016.



- [14] Y. Gao, W. Zhou, H. Ao, J. Chu, Q. Zhou, B. Zhou, K. Wang, Y. Li, and P. Xue, "A novel optimal joint resource allocation method in cooperative multicarrier networks: theory and practice," *Sensors*, Apr. 2016.
- [15] G. Huang, L. Luo, G. Zhang, P. Yang, D. Tang, and J. Qin, "QoS-driven jointly optimal subcarrier pairing and power allocation for decode-and-forward OFDM relay systems," *Wireless Pers. Commun.*, vol. 71, Issue 3, pp. 1597-1618, Aug. 2013.
- [16] X. Li, Q. Zhang, G. Zhang, M. Cui, L. Yang and J. Qin "Joint resource allocation with subcarrier pairing in cooperative OFDM DF multi-relay networks," *IET Commun.*, vol. 17, no. 5, pp. 872-875, May 2013.
- [17] Y. Li, W. Wang, J. Kong, and M. Peng, "Subcarrier pairing for amplify-and-forward and decode-and-forward OFDM relay links," *IEEE Commun. Lett.*, vol. 13, no. 4, pp. 209-211, Apr. 2009.
- [18] W. Dang, M. Tao, H. Mu, and J. Huang, "Subcarrier-pair based resource allocation for cooperative multi-relay OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1640-1649, May 2010.
- [19] W. C. Pao and Y. F. Chen, "Reduced complexity subcarrier allocation schemes for DFT-precoded OFDMA uplink systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2701-2706, Sep. 2010.
- [20] K. Singh and M.-L. Ku, "Toward green power allocation in relay-assisted multiuser networks: a pricing-based approach," *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2470- 2486, May 2015.
- [21] K. Singh, M.-L. Ku, and J.-C. Lin, "Optimal energy-efficient power allocation for multiuser relay networks," in *Proc. IEEE VTC Spring*, May 2014.
- [22] E. Bedeer, A. Alorainy, M. J. Hossain, O. Amin, and M.-S. Alouine, "Fairness-aware energy-efficient resource allocation for AF co-operative OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 12, pp. 2478-2492, Dec. 2015.
- [23] R. A. Loodaricheh, S. Mallick, and V. K. Bhargava, "Energy-efficient resource allocation for OFDMA cellular networks with user cooperation and QoS provisioning," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6132-6146, Nov. 2014.
- [24] O. Amin, E. Bedeer, M. H. Ahmed, and O. A. Dobre, "Energy efficiency-spectral efficiency tradeoff: a multiobjective optimization approach," *IEEE Trans. Veh. Technol.*, vol. 65, no. 4, pp. 1975-1981, Apr. 2016.
- [25] C. C. Zarakovitis, Q. Ni, and J. Spiliotis, "Energy efficient green wireless communication systems with imperfect CSI and data outage," *IEEE J. Sel. Areas Commun.*, vol. pp. no.99, Aug. 2016.
- [26] C. C. Zarakovitis and Q. Ni, "Maximizing energy efficiency in multiuser multicarrier broadband wireless systems: convex relaxation and global optimization techniques," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5275-5286, Jul. 2016.
- [27] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback self-interference in full-duplex MIMO relays," *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 5983-5993, Dec. 2011.
- [28] D. Bharadia, E. McMillin, and S. Katti, "Full Duplex Radios," in *Proc. ACM SIGCOMM*, pp. 375-386, Aug. 2013.
- [29] H. W. Kuhn, "The Hungarian method for the assignment problem," in *50 Years of integer programming 1958-2008.*, Springer Berlin Heidelberg, pp. 29-47, 2010.
- [30] S. Boyd and L. Vandenberghe, "Convex optimization," Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [31] H. Zhang, Y. Liu, and M. Tao, "Resource allocation with subcarrier pairing in OFDMA two-way relay networks," *IEEE Wireless Commun. Lett.*, vol. 1, no. 2, pp. 61-64, Jan. 2012.
- [32] A. Sard, "Linear approximation," *Amer. Math. Soc.*, Providence, R.I., 1963.
- [33] 3GPP, TR 36.819 (V9.0.0), "Further advancement for E-UTRA physical layer aspects (Release 9)," Mar. 2010.



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